

Math Virtual Learning

AP Statistics

April 20th, 2020

Lesson: April 20th, 2020

Objective:

- 1. Students will understand Type 1 & Type 2 errors & know how to reduce the probability of each error occurring in a significance test**
- 2. Students will understand the connection between Type 1 & Type 2 errors and power.**

Type I & Type II Errors in Significance Tests

Type I Error	The null hypothesis, H_0 , is actually true, but you have rejected it and concluded the alternative hypothesis, H_a , instead.
Type II Error	The alternative hypothesis, H_a , is actually true, but you have failed to reject the null, H_0 .

Review #1

Consider the scenario in which a cereal company claims that 20% of all its cereal boxes contain a voucher for a free DVD rental. A group of students believes the company is cheating and the proportion of all boxes with the vouchers is less than 0.20. They decide to collect some data to perform a test of significance with the following hypotheses.

$$H_o : p = 0.20$$

$$H_A : p < 0.20$$

where p = the proportion of all boxes with the voucher

Explain what a Type I error is in the context of the problem in this handout.

Answer to Review #1

Type I Error	The null hypothesis, H_0 , is actually true, but you have rejected it and concluded the alternative hypothesis, H_a , instead.
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With a Type 1 Error, the null hypothesis H_0 is actually true. So in the cereal box problem, the company is in fact telling the truth that 20% of their cereal boxes contain a voucher.

However, our statistical evidence leads us to reject the null and instead conclude the alternative H_a , which means we conclude that the company is cheating and less than 20% of cereal boxes contain the voucher.

Review #2

Consider the scenario in which a cereal company claims that 20% of all its cereal boxes contain a voucher for a free DVD rental. A group of students believes the company is cheating and the proportion of all boxes with the vouchers is less than 0.20. They decide to collect some data to perform a test of significance with the following hypotheses.

$$H_o : p = 0.20$$

$$H_A : p < 0.20$$

where p = the proportion of all boxes with the voucher

Explain what a Type II error is in the context of the problem in this handout.

Answer to Review #2

Type II Error	The alternative hypothesis, H_a , is actually true, but you have failed to reject the null, H_0 .
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With a Type 2 Error, the alternative hypothesis H_a is actually true. So in the cereal box problem, the company is in fact cheating and less than 20% of their cereal boxes contain a voucher.

However, our statistical evidence leads us to *fail to reject* the null and H_0 conclude that the company is cheating when they really are.

Example

R9.3. Strong chairs? A company that manufactures classroom chairs for high school students claims that the mean breaking strength of the chairs that they make is 300 pounds. One of the chairs collapsed beneath a 220-pound student last week. You wonder whether the manufacturer is exaggerating the breaking strength of the chairs.

State the two hypotheses and make note of what would be concluded if each one were true.

State the Hypotheses:

$H_0: \mu = 300$ If true, company is telling the truth

$H_a: \mu < 300$ If true, company is lying + students could get injured

Example Continued

Describe a Type 1 and Type 2 error in this problem and determine which error would be more serious.

Type I Error

The H_0 is actually true but I rejected it + concluded the H_a . So I would conclude that the company is lying when they are not.

Type II Error

The H_a is actually true, but I fail to reject the H_0 . So I conclude that the chairs hold 300lbs when really they hold less than that. More students could get injured.

Would a Type I or Type II error be more serious? Explain.

Type II error because students might get injured.

You Try (on separate paper)

Awful Accidents: Slow response times by paramedics, firefighters, and policemen can have serious consequences for accident victims. In the case of life-threatening injuries, victims generally need medical attention within 8 minutes of the accident. Several cities have begun to monitor emergency response times. In one such city, the mean response times to all accidents involving life-threatening injuries last year was minutes. Emergency personnel arrived within 8 minutes after 78% of all calls involving life-threatening injuries last year. The city manager shares this information and encourages these first responders to “do better.” At the end of the year, the city manager selects an SRS of 400 calls involving life-threatening injuries and examines the response times.

State the two hypotheses and make note of what would be concluded if each one were true.

Check your hypotheses & Describe the errors

State the Hypotheses:

$H_0: \mu = 6.7$ If true, response time is the same $\rightarrow$ people's lives at risk

$H_a: \mu < 6.7$ If true, response time improved $\rightarrow$ more lives saved

Describe a Type 1 and Type 2 error in this problem and determine which error would be more serious.

Check your work.

Type I Error

H_0 is true, but we conclude H_a .
Response time stayed the same
but we conclude that it
improved.

Consequence $\rightarrow$ people's lives
will be at risk

Type II Error

H_a is true, but we conclude H_0 .
Response time improved but
we conclude it stayed the
same.

Consequence $\rightarrow$ more lives saved
+ city continues to take
steps to improve response times

Would a Type I or Type II error be more serious? Explain.

Type I error since it puts people's lives in danger.

So how can we reduce the likelihood that a **Type 1 error** will occur?

$P(\text{Type I Error}) = \text{the significance level } \alpha$

So, if the consequences of a Type I error are very bad, then before beginning the significance test you will want to set α at a lower value like .01 instead of .05.

Example:

If you complete a significance test at a significance level $\alpha = 0.10$, then the probability that you will reject the null hypothesis when it is actually true (Type 1 error) is 10%.

How is **Type 2** error related to power?

The probability a test of significance will lead to a Type II error is denoted by the Greek letter β

Since Power is the probability of rejecting the null hypothesis when it is false, and β is the probability of failing to reject the null hypothesis when it is false, these two quantities represent the only decisions that can be made when the null hypothesis is false. Thus, the following relationship holds:

$$\text{Power} + \beta = 1$$

$$\text{Therefore, } P(\text{Type II error}) = 1 - \text{power}$$
$$\beta$$

Example

Power: The CDC claims that 10% of patients who take a new drug for treating Alzheimer's disease will experience nausea. The drug manufacturer believes that less than 10% will experience nausea. To test this claim, a significance test is carried out of

$$H_0: p = .10$$

$$H_a: p < .10$$

You learn that the power of this test at the 5% significance level against the alternative is 0.64.

Explain what the power means in the context of the problem.

Power is the probability of rejecting the null hypothesis when it is false (which is what you hope to do in any significance test). So in this problem, there is a 64% chance that you will find statistically significant evidence to correctly **reject** the CDC's claim that 10% of patients will experience nausea and instead conclude **less than** 10% of people will experience nausea when taking this drug.

Example Continued

Power: The CDC claims that 10% of patients who take a new drug for treating Alzheimer's disease will experience nausea. The drug manufacturer believes that less than 10% will experience nausea. To test this claim, a significance test is carried out of

$$H_0: p = .10$$

$$H_a: p < .10$$

You learn that the power of this test at the 5% significance level against the alternative is 0.64.

What is the probability of getting a Type 1 and Type 2 error?

The probability of getting a Type 1 error is 5% since that is the significance level, or alpha α

The probability of getting a Type 2 error is **1 - power** = 1 - 0.64 or **.36**.

β

Check Your Understanding (on separate paper)

A large university provides housing for 10 percent of its graduate students to live on campus. The university's housing office thinks that the percentage of graduate students looking for housing on campus may be more than 10 percent. The housing office decides to survey a random sample of graduate students to test the following hypotheses

$$H_o : p = 0.10$$

$$H_A : p > 0.10$$

where p = the proportion of all graduate students that want on campus housing

Suppose they get information from 500 respondents.

1. **Pretend** that in fact 18% of all graduate students want on campus housing. Do you think (no numerical calculations needed—just intuition and reasoning) that the test of significance performed by the housing office would have **low power** or **high power**? Be sure to define what power is as part of your explanation.

Check Your Understanding (on separate paper)

2. In the context of this situation, what would it mean for the housing office to make a **Type I Error**? Be sure to define what a Type I error is as part of your explanation.
3. In the context of this situation, what would it mean for the housing office to make a **Type II Error**? Be sure to define what a Type II error is as part of your explanation.

Check Your Understanding Answers

1. **Pretend** that in fact 18% of all graduate students want on campus housing. Do you think (no numerical calculations needed—just intuition and reasoning) that the test of significance performed by the housing office would have **low power** or **high power**? Be sure to define what power is as part of your explanation.

Power is the probability that they reject H_0 if it is false. A high power means a high probability of successfully rejecting H_0 . The power would be high, because the true p is significantly greater than the value given by the null hypothesis. If the true $p = .105$, the power would be much lower because it would be less probable for the test of significance to determine that the null hypothesis was false. Given the 8% margin between real p and the p of the null hypothesis, the power would be high.

Check Your Understanding Answers

2. In the context of this situation, what would it mean for the housing office to make a **Type I Error**? Be sure to define what a Type I error is as part of your explanation.

A Type I error would be rejecting the null hypothesis when the null hypothesis is true. In this case, if $p = .1$ but the simple random sample of the 500 students indicated that $p = .18$ or some value greater than .1 (the H_a being true). This might cause the university to unnecessarily expand its housing policies because it thinks that the percentage of students looking for houses is greater than it actually is.

Check Your Understanding Answers

3. In the context of this situation, what would it mean for the housing office to make a **Type II Error**? Be sure to define what a Type II error is as part of your explanation.

A Type II error would be failing to reject the null hypothesis when the null hypothesis is false. In this case, if $p > .1$ but the simple random sample of the 500 students did not give the housing office strong enough evidence to conclude that more than 10% of all graduate students need housing. This would cause the university to not expand its housing when in fact more housing for graduate students is needed.